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# Heavy Owa Operator of Generalized Octagonal Intuitionistic Fuzzy Numbers in Fuzzy Game

K.Selvakumari[1*] and S.Lavanya[2]
[1]*Department of Mathematics,*
*Vels Institute of Science, Technology and Advanced Studies, Chennai*
[2]*Department of Mathematics, Bharathi Women's College (Autonomous), Chennai*
*Corresponding Author: E-mail: selvafeb6@gmail.com*
*E-mail: lavanyaprasad1@gmail.com*

**Abstract-** A fuzzy game problem with pay offs as Generalized Octagonal intuitionistic fuzzy number (GOIFN) has been considered. In this paper a method to solve such a game problem using Heavy ordered weighted averaging (OWA) operator is introduced. In this method the GOIFN values of the payoff matrix collectively for each of the strategy of player A are aggregated by the Heavy OWA operator, and ranked by their membership and non-rnembership average indexes. Then the best strategy is chosen based on this ranking. The applicability of this rnethod is explained through a numerical exarnple.

**Keywords-** Octagonal Intuitionistic fuzzy numbers, fuzzy game matrix, ordered weighted averaging (OWA) operator.

## 1. INTRODUCTION

Game theory is very useful to study the ways in which strategic situations among each player getting outcomes with privilege to the preferences of those players. In real life situations, the players of games may not be able to find exactly the outcome of games with uncertain and asymmetric information between games players. In that case, the fuzzy sets initiated by Zadeh [9] is very much useful in game theory. Atanassov [1] initiated the concept of Intuitionistic fuzzy set (IF-set), Characterized by two functions represents the degree of membership value and the degree of non-membership value for each element.

The Ordered Weighted averaging (OWA) operator was developed by Yager [6] and one of its main functions is that it provides a parameterized family of aggregation operators that includes among others, the maximum, the minimum and the average criteria. Many researchers can be applied the OWA operator concept to fuzzy area. This paper focuses of the heavy ordered weighted averaging (HOWA) operator.

The HOWA operator [7] provides a parameterized family of aggregation operators that includes among others, the minimum, the OWA operator and the total operator. The heavy OWA operator has also been applied by incorporating fuzzy measures (Yager [9]), by incorporating fuzzy numbers (Merigo and Casanovas [3]). The ranking of Intuitionistic fuzzy numbers very useful with finding the solution of matrix games with payoffs represent as Intuitionistic fuzzy numbers.

## 2. PRELIMINARIES

### 2.1 Fuzzy set

Let X be a non-empty set. A fuzzy set A drawn from X can be defined as A= $\{\langle x, \mu_A(x)\rangle : x \in X\}$ where $\mu_A(x): X \to [0,1]$ is the membership function of the fuzzy set A.

### 2.2 Intuitionistic fuzzy set

Let X be a non-empty set. An Intuitionistic fuzzy set is of the form A= $\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A(x), \nu_A(x): X \to [0,1]$ is the degree of membership and degree of non-membership value of the element $\in X$ .

### 3.Generalized Octagonal Intuitionistic Fuzzy Number (GOIFN)

A GOIFN $\tilde{A}$ = ([$a_1$, $a_2$, $a_3$, $a_{4,}$ $a_5$, $a_6$, $a_7$, $a_8$]; $w_{\tilde{a}}$ , $u_{\tilde{a}}$] is a special Intuitionistic fuzzy set on a set of real number $\Re$, the membership function and non-membership function are defined as follows:

$$\mu_{\tilde{A}}(x)\begin{cases} 0 & for x < a_1 \\ k_1\left(\dfrac{x-a_1}{a_2-a_1}\right) & for a_1 \le x \le a_2 \\ k_1 & for a_2 \le x \le a_3 \\ k_1 + (w_{\tilde{a}} - k_1)\left(\dfrac{x-a_3}{a_4-a_3}\right) & for a_3 \le x \le a_4 \\ w_{\tilde{a}} & for a_4 \le x \le a_5 \\ k_1 + (w_{\tilde{a}} - k_1)\left(\dfrac{a_6-x}{a_6-a_5}\right) & for a_5 \le x \le a_6 \\ k_1 & for a_6 \le x \le a_7 \\ k_1\left(\dfrac{a_8-x}{a_8-a_7}\right) & for a_7 \le x \le a_8 \\ 0 & for x > a_8 \end{cases}$$

$$\vartheta_{\tilde{A}}(x) = \begin{cases} 1 & for x < a_1 \\ 1+(1-k_2)\left(\dfrac{a_1{'}-x}{a_2{'}-a_1}\right) & for a_1 \le x \le a_2 \\ k_2 & for a_2 \le x \le a_3 \\ k_2+(k_2-u_{\tilde{a}})\left(\dfrac{a_3{'}-x}{a_4{'}-a_3{'}}\right) & for a_3 \le x \le a_4 \\ 0 & for a_4 \le x \le a_5 \\ u_{\tilde{a}}+(k_2-u_{\tilde{a}})\left(\dfrac{x-a_5{'}}{a_6{'}-a_5}\right) & for a_5 \le x \le a_6 \\ k_2 & for a_6 \le x \le a_7 \\ k_2+(1-k_2)\left(\dfrac{x-a_7{'}}{a_8{'}-a_7{'}}\right) & for a_7 \le x \le a_8 \\ 1 & for x > a_8 \end{cases}$$

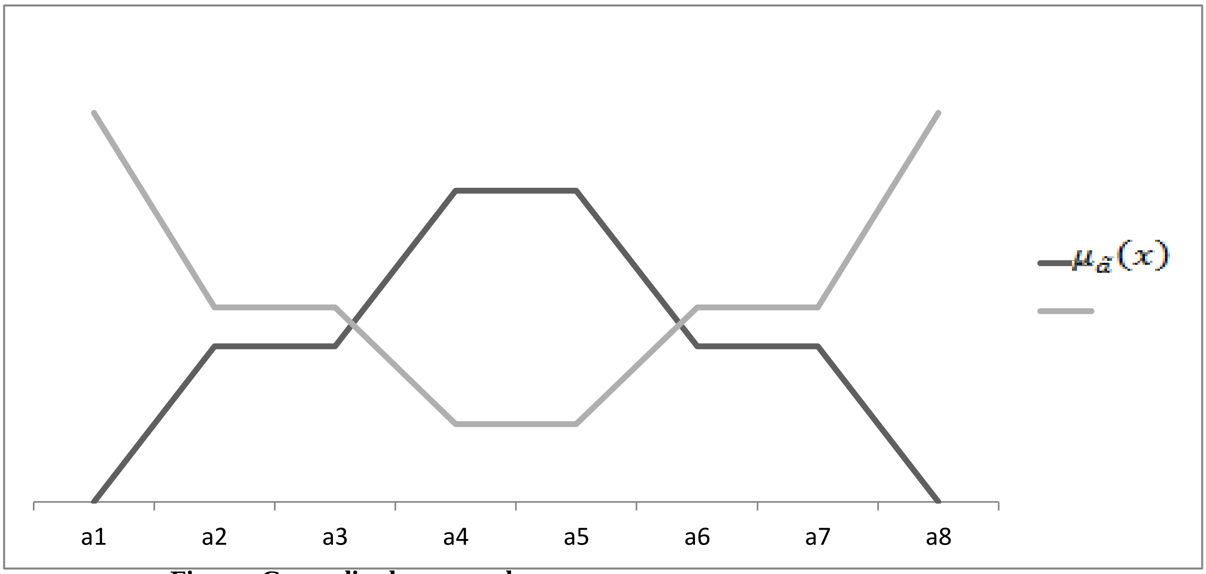


**Figure: Generalised octagonal intutionistic fuzzy number**

**4 .Arithmetic Operations of GOIFN**

Let $\tilde{a} = ([a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8];\ w_{\tilde{a}}\ ,\ u_{\tilde{a}})$ and $\tilde{b} = ([b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8]; w_{\tilde{b}}\ ,\ u_{\tilde{b}})$ be a two OIFN

The arithmetic operations for GOIFN are

1. $\tilde{a}+\tilde{b} = ([a_1+ b_1,\ a_2+ b_2,\ a_3+ b_3,\ a_4+ b_4,\ a_5+ b_5,\ a_6+ b_6,\ a_7+ b_7,\ a_8+ b_8]; w_{\tilde{a}} \wedge w_{\tilde{b}}\ , u_{\tilde{a}} \vee u_{\tilde{b}})$
2. $\tilde{a}-\tilde{b} = ([a_1- b_8,\ a_2- b_7,\ a_3- b_6,\ a_4- b_5,\ a_5- b_4,\ a_6- b_3,\ a_7- b_2,\ a_8- b_1]; w_{\tilde{a}} \wedge w_{\tilde{b}}\ , u_{\tilde{a}} \vee u_{\tilde{b}})$

**5. Ranking of average index of the membership and non-membership for**

$R_{\mu}(\tilde{a}) = w_{\tilde{a}}\left[\frac{k_1(a_1+a_2+a_7+a_8)+(1-k_1)(a_3+a_4+a_5+a_6)}{4}\right]$

$$= (1 - R_{\vartheta}(\tilde{a}))\left[\frac{k_2(a_1+a_2+a_7+a_8)+(1-k_2)(a_3+a_4+a_5+a_6)}{4}\right]$$

The average Index of the membership $\mu_{\tilde{a}}(x)$ , the non-membership $\vartheta_{\tilde{a}}$(x) for GOIFN are defined as

## 6. HEAVY OWA OPERATORS

The OWA operator initiated by Yager [7,8] in 1988 provides parameterized family of aggregation operators between the maximum and the minimum. Yager introduced an extension of the OWA operator called the heavy OWA operator which allows the weighting vector to range between the OWA operator and the total operator. In some situations, available information is independent from each other, such cases need this operator.

### 6.1 Definition

A Heavy OWA operator of dimension n is a mapping $HOWA: R^n \to R$ that has an associated weighting vector w of dimension n with $w_j \in [0,1]$ and $1 \le \sum_{j=1}^{n} w_j \le n$, such that

HOWA ($A_1$, $A_2$, …, $A_n$) = $\sum_{j=1}^{n} w_j\, a_{ij}$, where $a_{ij}$is the strategy of player A

HOWA ($B_1$, $B_2$, …, $B_n$) = $\sum_{j=1}^{n} w_j\, b_{ij}$, where $b_{ij}$is the strategy of player B

## 7. Heavy OWA Operators for Fuzzy Games with Payoff as GOIFN

$$\tilde{r}_{ij}^{(k)} = \frac{a_{ij}^{(k)} - \min a_{ij}^{(1)})}{\max_i a_{ij}^{(8)} - \min_i a_{ij}^{(1)}} \text{, for j=1, 2...n,}$$

$$\tilde{r}_{ij}^{(k)} = \frac{\max a_{ij}^{(8)} - a_{ij}^{(9-k)}}{\max_i a_{ij}^{(8)} - \min_i a_{ij}^{(1)}} \text{, for i=1, 2…m}$$

*w*here k=1, 2, 3, 4, 5, 6, 7, 8

## 8. PROCEDURE FOR HEAVY OWA OPERATORS FOR INTUITIONISTIC FUZZY GAMES

**Step (1)** The given payoff-matrix can be converted into Normalized pay off matrix according to the equation.

**Step (2)** Use the Heavy OWA operator of GOIFN to aggregate normalized decision matrix for player I. Let us assume that the weighting vector is W= ($w_1$, $w_2$… $w_n$), then the collective overall fuzzy number of player I is H $(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \sum_{j=1}^{n} w_j\, a_{ij}$

$$= (\left[\sum_{j=1}^{n} w_j a_{1j}, \sum_{j=1}^{n} w_j\, a_{2j}, \sum_{j=1}^{n} w_j\, a_{3j}, \dots . \sum_{j=1}^{n} w_j\, a_{nj}\right]$$

; $\min_{1 \le i \le n} w_{\tilde{a}_i}, \max_{1 \le i \le n} u_{\tilde{a}_i})$

Similarly, the collective overall fuzzy number of player II is

$$\text{H } (\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n) = \sum_{j=1}^{n} w_j\, a_{ij}$$

**Company B**

$$\begin{array}{c} Media \\ Radio \\ T.V \end{array} \left( \begin{array}{ccc} \overset{Media}{([2,2.2,2.5,3,4,4.2,4.7,5]; 0.8,0.2)} & \overset{Radio}{\begin{pmatrix}[2,2.5,3,4,5,5.5,6,7]; \\ 0.6,0.4\end{pmatrix}} & \begin{pmatrix}[1,1.5,2 \\ 0\end{pmatrix} \\ \begin{pmatrix}[1,1.5,2,3,6,6.2,6.5,7]; \\ 0.7,0.2\end{pmatrix} & \begin{pmatrix}[3,3.3,3.5,4,5,5.1,5.5,6]; \\ 0.9,0.1\end{pmatrix} & \begin{pmatrix}[5,5.5,6, \\ 0\end{pmatrix} \\ ([3,3.1,3.5,4,5,6,7,8]; 0.5,0.4) & ([2,2.5,2.8,3,4,4.5,5,6]; 0.8,0.2) & \begin{pmatrix}[4,5,5.5 \\ 0\end{pmatrix} \end{array} \right.$$

**Company A**

Let us consider a matrix game with payoffs of GOIFN and two sets of strategies for player I and player II. The Payoff matrix for Player I is given by $\mathcal{M} = (\tilde{a}_{ij})_{mxn}$, where

$\tilde{a}_{ij}$ = $([a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]; w_{\tilde{a}_i}, u_{\tilde{a}_i})$, (i=1, 2…m; j=1, 2…n) is a GOIFN.

The heavy OWA operators for GOIFN are used to solve the corresponding Game problem. Assume that the payoff matrix is characterized by GOIFN

$\tilde{a}_{ij}$=($[a_{ij}^{(1)}, a_{ij}^{(2)}, a_{ij}^{(3)}, a_{ij}^{(4)}, a_{ij}^{(5)}, a_{ij}^{(6)}, a_{ij}^{(7)}, a_{ij}^{(8)}]$ ,$w_{\tilde{a}_i}$ , $u_{\tilde{a}_i}$)

The Normalized payoff matrix for player I and II is

$$= (\left[\sum_{i=1}^{n} w_j a_{i1}, \sum_{j=1}^{n} w_j\, a_{i2}, \sum_{j=1}^{n} w_j\, a_{i3}, \dots . \sum_{j=1}^{n} w_j\, a_{in}\right]$$

; $\min_{1 \le i \le n} w_{\tilde{a}_i}, \max_{1 \le i \le n} u_{\tilde{a}_i})$

**Step (3)** For each player, calculate membership, non-membership average indexes for its collective overall Fuzzy number H($A_i$) and H($B_i$), then the ranking orders of Players are obtained using ranking method .

## 9. NUMERICAL EXAMPLE

**Game Problem with Payoff as Generalized Octagonal Intuitionistic Fuzzy Numbers**

Suppose that two companies are competing for business whatever company A gains company B loses. The following problem shows advertising strategies of both companies and the utilities to company A for various market shares in percentage. The company A and B are represented as players I and II respectively.

**Best Strategy for player I**

**Step (1)**

According to equation

$\tilde{r}_{ij}^{(k)} = \frac{a_{ij}^{(k)} - \min a_{ij}^{(1)})}{\max_i a_{ij}^{(8)} - \min_i a_{ij}^{(1)}}$ , for j= 1,2,..n ,the original game matrix $\mathcal{M}=(\tilde{a}_{ij})_{mxn}$ can be converted into normalized matrix

**Step (2)**

Use the Heavy OWA operator of GONFN to aggregate normalized matrix for player I. Let us assume that the weighting vector is W= ($w_1, w_2 \dots w_n$), then the collective overall GOIFN of player I is H $(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \sum_{j=1}^{n} w_j\, a_{ij}$

$$= ([\sum_{j=1}^{3} w_j a_{1j}, \sum_{j=1}^{3} w_j\, a_{2j}, \sum_{j=1}^{3} w_j\, a_{3j}]$$

$; \min_{1\le i\le n} w_{\tilde{a}_i}, \max_{1\le i\le n} u_{\tilde{a}_i})$

The weight of strategies are W= (0.15, 0.4, 0.45).

The heavy ordered weighted value for $A_1$ is

$H(A_1)= 0.1\,([0.14,0.17,0.21,0.29,0.43,0.46,0.53,0.57]; 0.8,0.2)$
$+0.4*([0,0.1,0.2,0.4,0.6,0.7,0.8,1]; 0.6,0.4)$
$+0.45*([0,0.06,0.13,0.25,0.63,0.69,0.75,0.88]; 0.6,0.3)$

= ([0.021, 0.093, 0.171, 0.317, 0.589, 0.66, 0.738, 0.882] ;

0.6, 0.4)

The heavy ordered weighted value for $A_2$ is

$H(A_2)=0.15*([0,0.07,0.14,0.29,0.71,0.74,0.79,0.89]; 0.7,0.2)$
$+0.4*([0.2,0.26,0.3,0.4,0.6,0.62,0.7,0.8]; 0.9,0.1)+0.45*([0.5,0.56,0.63,0.75,0.88,0.9,0.94,1]; 0.7,0.1)$

= ([0.305, 0.367, 0.425, 0.542, 0.743, 0.764, 0.822,

0.904]; 0.7, 0.2)

The heavy ordered weighted value for $A_3$ is

$H(A_3)=0.15*(([0.29,0.3,0.36,0.43,0.57,0.71,0.86,1]; 0.5,0.4)$
$+0.4*([0,0.1,0.16,0.2,0.4,0.5,0.6,0.8]; 0.8,0.2)+0.45*([0.38,0.5,0.56,0.63,0.75,0.81,0.88,1]; 0.6,0.4)$

= ([0.215, 0.31, 0.37, 0.429, 0.584, 0.672, 0.765,

0.92]; 0.5, 0.4)

**Step (3)**

For each Strategy, calculate the membership and non-membership average indexes for its collective overall GOIFN H ($A_i$), then the ranking order of strategies are obtained using the ranking method .

$R_\mu$[H ($A_1$)] = 0.26 ; $R_\vartheta$[H ($A_1$)] = 0.26

$R_\mu$[H ($A_2$)] = 0.43 ; $R_\vartheta$[H ($A_2$)] = 0.49

$R_\mu$[H ($A_3$)] = 0.27 ; $R_\vartheta$[H ($A_3$)] = 0.32

Then the ranking order of the strategies for Player I according to the membership and non-membership average indexes is given by $A_2 > A_3 > A_1$, Hence the best strategy for player I is $A_2$.

**Best Strategy for player II**

**Step(1)**

According to equation

$\tilde{r}_{ij}^{(k)} = \frac{\max a_{ij}^{(8)} - a_{ij}^{(9-k)}}{\max_i a_{ij}^{(8)} - \min_i a_{ij}^{(1)}}$ , for$i = 1,2, \dots m$, the original game matrix $\mathcal{M} = (\tilde{a}_{ij})_{mxn}$ can be converted into normalized matrix

**Step (2)**

Use the Heavy OWA operator of GOIFN to aggregate normalized matrix for player II. Let us assume that the weighting vector is W= ($w_1, w_2 \dots w_n$), then the collective overall GOIFN of player II is H $(\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n) = \sum_{j=1}^{n} w_j\, a_{ij}$

$$= (\left[\sum_{i=1}^{3} w_j a_{i1}, \sum_{j=1}^{n} w_j\, a_{i2}, \sum_{j=1}^{3} w_j\, a_{i3}\right]$$

$; \min_{1\le i\le n} w_{\tilde{a}_i}, \max_{1\le i\le n} u_{\tilde{a}_i})$

The weight of strategies are W= (0.15,0.4,0.45).

The heavy ordered weighted value for $B_1$is

$H(B_1)=0.15*([0.43, 0.47,0.54,0.57,0.71,0.79,0.83,0.86]; 0.8,0.2)$

$0.4*([ ([0.25,0.31,0.35,0.38,0.75,0.88,0.94,1]; 0.7,0.2) )$

$+0.45*$
$([0.14,0.29,0.43,0.57,0.71,0.79,0.83,0.86]; 0.5,0.4)$

= ([0.225,0.325,0.415,0.494,0.726, 0.826,0.874, 0.916]; 0.5,0.4

The heavy ordered weighted value for $B_2$is

$H(B_2) =$

$0.15*$
$([0.14,0.29,0.36,0.43,0.57,0.71,0.79,0.86]; 0.6,0.4)$

$+0.4*($
$([0.38,0.44,0.49,0.5,0.63,0.69,0.71,0.75]; 0.9,0.1)$

$+0.45*$
$([0.43,0.57,0.64,0.71,0.86,0.89,0.93,1]; 0.8,0.2)$

= ([0.367,0.476,0.538, 0.584, 0.725, 0.783, 0.821, 0.879]; 0.6, 0.4)

The heavy ordered weighted value for $B_3$is

$H(B_3) =$

$0.15*($
$([0,0.14,0.21,0.29,0.71,0.86,0.93,1]; 0.6,0.3)$

$+0.4*([0,0.06,0.1,0.13,0.25,0.38,0.44,0.5]; 0.7,0.1)$

$+0.45 *$
$([0,0.14,0.21,0.29,0.43,0.5,0.57,0.71]; 0.6,0.4)$

= ([0, 0.108, 0.166, 0.226, 0.4, 0.506, 0.572, 0.670]; 0.6, 0.4)

**Step (3)**

For each Strategy, calculate the membership and non-membership average indexes for its collective overall GOIFN $H(B_i)$, then the ranking order of strategies are generated according to the ranking method of GOIFN

$R_\mu[H(B_1)] = 0.30$ ; $R_\vartheta[H(B_1)] = 0.36$

$R_\mu[H(B_2)] = 0.39$ ; $R_\vartheta[H(B_2)] = 0.39$

$R_\mu[H(B_3)] = 0.20$ ; $R_\vartheta[H(B_3)] = 0.20$

Then the ranking order of the strategies for Player B according to the membership and non-membership average indexes is given by $B_2>B_1>B_3$, Hence the best strategy for player II is $B_3$.

The best strategy for player I and II are $A_2$ and$B_3$ respectively.

## 3. CONCLUSION

In this work, we have considered a fuzzy game problem with Payoffs as Generalized Octagonal Intuitionistic fuzzy numbers (GOIFN).We have introduced a method to solve such a game problem using Heavy ordered weighted averaging (OWA) operator and explained it through a numerical example.

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